Reply to comments by Au

1997 J. Phys. A: Math. Gen. 30 1789

(http://iopscience.iop.org/0305-4470/30/5/039)

J. Phys. A: Math. Gen. **30** (1997) 1789–1790. Printed in the UK PII: S0305-4470(97)76522-3

COMMENT

Reply to comments by Au

T K Nandi†, P K Bera, M M Panja and B Talukdar
Department of Physics, Visva-Bharati University, Santiniketan 731 235, India

Received 8 July 1996

Abstract. We reply to the comments by Au on the Dalgarno–Lewis method as a perturbation theory.

Following an early work by Aharonov and Au [1], the logarithmic perturbation technique (LPT) has been extensively studied by the second named author. The main problem appears to have been solved more or less satisfactorily leaving rather little opening for further investigation. In dealing with the Dalgarno–Lewis (DL) method [2], we did not have any desire either to plagiarize or to paraphrase Au. On the other hand, we tried to provide a significant addendum for the role of DL technique in perturbation theoretic calculations by using the ansatz [3] given by (5) of our paper [4]. The physical interpretation sought by us for the function $S(x)$ and the subsequent adaptation of the method for scattering problems are expected to be quite important to initiate more detailed works. In the following we judiciously use some of the comments made by Au [5] to look for further justification for our claim in [4].

The equation written by Kim and Sukhatme [6] to calculate the first-order correction to the unperturbed wavefunction $\psi_n^{(0)}$ is given by

$$\psi_n^{(0)} f_n^{(1)\prime\prime} + \psi_n^{(0)\prime} f_n^{(1)\prime} = (h - E_n^{(1)})\psi_n^{(0)} \tag{1}$$

where only $f_n (= f_n^{(0)} + \lambda f_n^{(1)} + \lambda^2 f_n^{(2)} + \cdots)$ is assumed to respond to the perturbation h. The equation in (1) can easily be converted to that of $f_n^{(1)}\psi_n^{(0)}$ giving the well known inhomogeneous equation of Dalgarno and Lewis. Similar results can be found for all orders of λ. Thus, the approach of Kim and Sukhatme is a simple variant of the DL method in one dimension and, at the same time, a special instance of the more general result discussed by Au [7] for multidimensional systems. But the point of interest is that the observed correspondence identifies the singularities of $f_n^{(1)}(x)$ at zeros of $\psi_n^{(0)}(x)$ without reference to any specific example. Admittedly, this is an added realism of the DL method.

In a relatively recent work, Au *et al* [8] extended the logarithmic perturbation expansion to deal with the singularity problem of excited bound states in one dimension. The development was based on a peculiar attention to the complex combination of regular and irregular solutions of the Schrödinger equation. It remains an interesting curiosity to relate this approach to the somewhat antique but seminal work of Zeldovich [9]. In the work of Zeldovich, a form of the perturbation theory was developed by using Lagrange's method of variation of parameters [10] in which correction to the unperturbed wavefunction

† E-mail address: root@vbharat.ernet.in

0305-4470/97/051789+02$19.50

and quantization condition were obtained by imposing constraints on the coefficient of the irregular solution of the unperturbed problem. The treatment presented by Au [8], in principle, does not differ from that of Zeldovich. Particularly, one can show that up to first-order correction, (2.17) of Au corresponds to the bound-state version of (7) in Zeldovich. As expected, in both approaches, corrections to the unperturbed energy levels are obtained in the same algebraic form.

As with excited bound states, the use of LPT will be complicated by the nodal problem of the regular solution of the unperturbed radial Schrödinger equation. But the DL method does not exhibit any such problem. Therefore, the continuum state perturbation theory derived by us [4] may turn out to be quite straightforward for real applications.

References

[1] Aharonov Y and Au C K 1979 *Phys. Rev. Lett.* **42** 1582
[2] Dalgarno A and Lewis J T 1955 *Proc. R. Soc.* **233** 70
[3] Goldstein H 1985 *Classical Mechanics* (New Delhi: Addison-Wesley/Narosa)
[4] Nandi T K, Bera P K, Panja M M and Talukdar B 1996 *J. Phys. A: Math. Gen.* **29** 1101
[5] Au C K 1997 *J. Phys. A: Math. Gen.* **30** 1785
[6] Kim I W and Sukhatme U P 1992 *J. Phys. A: Math. Gen.* **25** L647
[7] Au C K 1984 *Phys. Rev.* A **29** 1034
[8] Au C K, Chan K L, Chow C K, Chu C S and Young K 1991 *J. Phys. A: Math. Gen.* **24** 3837
[9] Zel'Dovich Ia B 1957 *Sov. Phys–JETP* **4** 942
[10] Ince E L 1926 *Ordinary Differential Equation* (New York: Dover)